CH: 12 ELECTROSTATICS

1) Number of lines / Tubes of forces:

$$N = \frac{q}{\epsilon_0 K}$$

2) Electric field intensity at a point outside a charged conducting sphere:

$$E = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{Q}{r^2} = \frac{\sigma R^2}{\epsilon_0 K r^2}$$

3) Electric field intensity at a point close & outside a charged conducting cylinder:

$$E = \frac{q}{2\pi\epsilon_0 K r}$$

4) Electric field intensity at a point close & outside surface of a charged conductor:

$$E = \frac{\sigma}{\epsilon} = \frac{\sigma}{\epsilon_0 K}$$

5) Electric field intensity at a point uniformly due to charged infinite plane sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

6) Mechanical force per unit area of charged conductor:

$$f = \frac{\sigma^2}{2\epsilon_0 K} = \frac{1}{2}\epsilon_0 K E^2$$

7) Energy density:

$$\frac{dU}{dV} = \frac{1}{2}\epsilon_0 K E^2 = \frac{\sigma^2}{2\epsilon_0 K}$$

8) Capacity of conductor / condenser:

$$C = \frac{Q}{V}$$

9) Capacity of parallel plate condenser:

$$C = \frac{A\epsilon_0 K}{d}$$

10) Energy stored in conductor:

$$U = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

11) Series combination of condensers:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

12) Parallel combination of condensers:

$$C_p = C_1 + C_2 + C_3 + \cdots$$

13) Surface charge density:

$$\sigma = \frac{Q}{A}$$

14) Electric intensity: (Electric field)

$$E = \frac{\text{Potential difference}}{\text{Distance}} = \frac{V}{d}$$

15) $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \; Nm^2/C^2$, $\varepsilon_0 = 8.85 \times 10^{-12} \; C^2/Nm^2$ or F/m

16) Gauss' theorem:

$$\left.\begin{array}{l}\text{T.N.E.I. over closed} \\ \text{surface of any shape}\end{array}\right\} = q_1 + q_2 + q_3 + \cdots = \Sigma q$$

17) Electric flux thro' area S: (Gauss' theorem)

$$\phi = \vec{E} \cdot \vec{S} = ES\cos\theta = \frac{q}{K\varepsilon_0}$$

18) Electric potential: $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

19) Dielectric constant: $K = \frac{C}{C_{air}}$

(C = Capacity with dielectric, C_{air} = w/o dielectric ie. air)

20) Time constant of RC circuit: $\tau = RC$

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1) An electron experiences a force equal to its weight, when placed at a point in an electric field. What is the intensity of the electric field at that point? [$m_e = 9 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C]

a) 10×10^{-8} N/C b) 5.5×10^{-11} N/C

c) 15×10^{11} N/C d) 2×10^{-11} N/C

⇒ (b) $F = mg$ & $F = qE$

$\therefore qE = mg$

$\therefore E = \frac{mg}{q} = \frac{9 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}}$

$= 5.5 \times 10^{-11}$ N/C

(2) A surface $\vec{S} = 10j$ is kept in an electric field of $\vec{E} = 3i + 5j + 6k$. How much electric flux will come out through the surface?

a) 30 units b) 40 units

c) 50 units d) 60 units

⇒ (c) Flux is $\phi = \vec{E} \cdot \vec{S}$

$= (3i + 5j + 6k) \cdot (10j)$

$= (3)(0) + 5(10) + 6(0)$

$= 50$ units

(3) A point P is at a distance r from a point charge q. If at point P, the electric potential is 300V, and the electric field intensity is 75 V/m, then the distance of P from the point charge is —

a) 2m b) 4m c) 8m d) 10m

⇒ (b) Electric field intensity, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

& Electric potential, $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

$\therefore \frac{V}{E} = \frac{q/r}{q/r^2} = r$

$\therefore r = \frac{V}{E} = \frac{300}{75} = 4$ m

(4) If n drops of equal size & same potential V merge into a big drop, the new potential of the big drop will be —

a) $V^{n/3}$ b) $n^{1/3} V$ c) nV d) $n^{2/3} V$

⇒ (d) Let r = Radius of small drop
& R = —"— big drop.

$\therefore V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$ & $V_{Big} = n \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$

Now, [Volume of small drop] $\times n$ = [Volume of Big drop]

$$\therefore \frac{4}{3}\pi r^3 \times n = \frac{4}{3}\pi R^3$$

$$\therefore n r^3 = R^3$$

Taking cube root

$$\therefore R = n^{1/3} r$$

$$\therefore V_{Big} = \frac{n}{4\pi\epsilon_0} \cdot \frac{q}{n^{1/3} r} = \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}\right] \cdot \frac{n}{n^{1/3}}$$

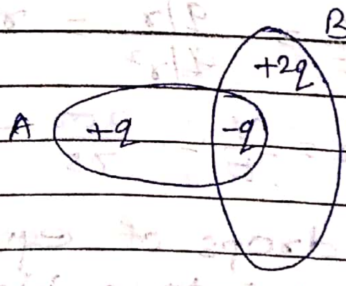
$$= \frac{nV}{n^{1/3}} = n^{1-1/3} \cdot V$$

$$= n^{2/3} \cdot V$$

5. The potential of a spherical conductor of radius 5 cm is 10 V. What is the potential at the centre of the sphere?
a) 2V b) 10V c) zero d) 50V

⇒ (b) The electric potential at any point inside a charged sphere is the same as that on the surface: ie. 10 V.

6. What is T.N.E.I. through the surfaces A & B?
a) $(q, 0)$
b) $(q, 2q)$
c) $(-q, +q)$
d) $(0, q)$



⇒ (d) T.N.E.I. $= \Sigma q$ (By Gauss's Theorem)

For A, charge enclosed $= q - q = 0$

For B, charge enclosed $= 2q - q = q$

$\therefore$ T.N.E.I. are $(0, q)$

7) A point charge produces an electric flux of -1×10^3 Nm^2/C to pass through a ~~gua~~ gaussian surface of radius 10 cm centred on the charge. What is the value of the point charge?

a) 7.85×10^{-6} C b) 8.85×10^{9} C

c) -8.85×10^{-9} C d) 4.425×10^{-9} C

⇒ (c) As per Gauss' theorem,

$$\phi = \frac{q}{K\epsilon_0} = \frac{q}{\epsilon_0}$$

$$\therefore q = \phi . \epsilon_0 = -1 \times 10^3 \times 8.85 \times 10^{-12}$$

$$= -8.85 \times 10^{-9} \text{ C}$$

8) A charge of q μC is placed at the centre of a cube of side 0.1 m. The electric flux diverging from each face of the cube will be ___

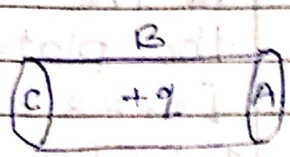
a) $\frac{q \times 10^{-6}}{\epsilon_0}$ Vm b) $\frac{q \times 10^{-6}}{6\epsilon_0}$ Vm

c) $\frac{6q \times 10^{-6}}{\epsilon_0}$ Vm d) $\frac{q \times 10^{-9}}{\epsilon_0}$ Vm

⇒ (b) $\phi = \frac{q}{K\epsilon_0} = \frac{q}{\epsilon_0}$ $q = q\,\mu C = q \times 10^{-6}$ C

$\therefore \phi = \frac{q \times 10^{-6}}{6\epsilon_0}$ $\left\{ \begin{array}{l} \because q = q\,\mu C = q \times 10^{-6} C \\ \text{\& there are six surfaces of cube.} \end{array} \right.$

9) A hollow cylinder has a charge +q coulomb within it. ϕ is the electric flux associated with the curved surface B.



What is the flux linked with the plane surface A?

a) $\frac{q}{2\epsilon_0}$ b) $\frac{\phi}{2}$

c) $\frac{q}{\epsilon_0} - \phi$ d) $\frac{1}{2}\left[\frac{q}{\epsilon_0} - \phi\right]$

⇒ (d) According to Gauss' theorem

$$\phi = \frac{q}{\epsilon_0} \quad \text{—— (1)}$$

Let ϕ_A, ϕ_B & ϕ_C are fluxes of A, B & C.

$$\therefore \phi = \phi_A + \phi_B + \phi_C$$

$$= \phi_A + \phi_B + \phi_A \quad \left\{ \because \phi_A = \phi_C \text{ Due to symmetry} \right.$$

$$\therefore \phi = 2\phi_A + \phi_B \quad \text{—— (2)}$$

From eqⁿ (1) & (2) ⇒

$$\frac{q}{\epsilon_0} = 2\phi_A + \phi_B$$

$$\therefore 2\phi_A = \frac{q}{\epsilon_0} - \phi_B$$

$$\therefore \phi_A = \frac{1}{2}\left[\frac{q}{\epsilon_0} - \phi\right] \quad \left\{ \because \phi_B = \phi \text{ ---Given} \right.$$

(10) A metal plate of area $2m^2$ is charged with 12×10^{-6} C. The surface density of charge is —

a) $3 \mu C/m^2$ b) $8 \mu C/m^2$ c) $6 \mu C/m^2$ d) $4 \mu C/m^2$

⇒ (c) Surface density,

$$\sigma = \frac{Q}{A} = \frac{12 \times 10^{-6}}{2} = 6 \times 10^{-6} \ C/m^2$$

$$= 6 \ \mu C/m^2$$

(11) A metal plate of surface area $2m^2$ is charged with $\sqrt{8.85}$ μC. What is the mechanical force acting on the plate if it is kept in air?

$[\epsilon_0 = 8.85 \times 10^{-12} \ C^2/Nm^2]$

a) 0.5 N b) 2 N c) 1 N d) 1.5 N

⇒ (c) Mechanical force per unit area is

$$f = \frac{\sigma^2}{2\epsilon_0 K}$$

∴ Total mechanical force, $F = \frac{\sigma^2}{2\epsilon_0 K} \times A$.

$$\therefore F = \frac{(\sqrt{8.85 \times 10^{-6}})^2}{2 \times 8.85 \times 10^{-12} \times 1} \times 2$$

$$= \frac{8.85 \times 10^{-12}}{2 \times 8.85 \times 10^{-12}} \times 2$$

$$= 1\ N$$

(12) A condenser has a capacity of $2\,\mu F$ & is charged to a potential of 50V. The energy stored in it is ___

a) 25×10 erg b) 25×10^3 erg
c) 25×10^5 erg d) 25 J

⇒ (b) Energy stored in condenser is

$$U = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \times 2 \times 10^{-6} \times (50)^2 = 2500 \times 10^{-6}\ J$$

$$= 2500 \times 10^{-6} \times 10^7\ \text{erg} \quad [1\ J = 10^7\ \text{erg}]$$

$$= 25 \times 10^3\ \text{erg}$$

(13) If the energy stored in a condenser of capacity $8\,\mu F$ is 4 J, what is the charge on the condenser?

a) 2×10^{-3} C b) 4×10^{-3} C
c) 6×10^{-3} C d) 8×10^{-3} C

⇒ (d) $U = 4\ J$, $C = 8\,\mu F = 8 \times 10^{-6}\ F$, $Q = ?$

$$U = \frac{1}{2} \cdot \frac{Q^2}{C}$$

$$\therefore Q^2 = 2UC$$

$$= 2 \times 4 \times 8 \times 10^{-6}$$

$$\therefore Q^2 = 64 \times 10^{-6}$$

$$\therefore Q = 8 \times 10^{-3}\ C$$

(14) A parallel plate capacitance of $4 \mu F$ is having a charge of 0.5 C. What will be its capacity if the charge is increased to 1 coulomb?

a) $8 \mu F$ b) $4 \mu F$ c) $2 \mu F$ d) $16 \mu F$

⇒ (b) Capacity of parallel plate Capacitor

$$C = \frac{A \epsilon_0 K}{d}$$

Thus it is independent of charge.

$\because C = \frac{Q}{V}$, As charge is increased, V also increases & $\therefore$ C remains constant. $\therefore C = 4 \mu F$ only.

(15) A condenser is charged through a P.D. of 100 volts & acquires a charge of 0.1 C. When discharged, it would release an energy ——

a) 1 J b) 2 J c) 5 J d) 10 J

⇒ (c) Energy discharged = Energy stored

$$\therefore U = \frac{1}{2} QV$$

$$= \frac{1}{2} \times 0.1 \times 100 = 5 J$$

(16) The capacity of a parallel plate condenser is $12 \mu F$. What is the new capacity if the separation between the plates is doubled & the area is halved?

a) $1.5 \mu F$ b) $3 \mu F$ c) $6 \mu F$ d) $8 \mu F$

⇒ (b) $C = \frac{A \epsilon_0 K}{d}$

$$\& \; C' = \frac{(A/2) \epsilon_0 K}{(2d)} = \frac{1}{4} \cdot \frac{A \epsilon_0 K}{d} \quad \left\{ \begin{array}{l} \because d' = 2d \\ \& A' = \frac{A}{2} \end{array} \right.$$

$$= \frac{C}{4} = \frac{12}{4} = 3 \mu F$$

17) A capacitor is used to store 24 watt hour of energy at 1200 volt. What should be the capacitance of the capacitor ?

a) 120 mF b) 120 μF c) 24 μF d) 24 mF

⇒ (a) U = 24 watt hour = 24×60×60 J {1 hr = 60×60 s}

$U = \frac{1}{2}CV^2$

$\therefore C = \frac{2U}{V^2} = \frac{2\times24\times60\times60}{1200\times1200} = \frac{2\times24}{400}$

$= 12\times10^{-2}$ F $= 120\times10^{-3}$ F

= 120 mF

18) The charge on a 48 μF capacitor is increased from 0.1 C to 0.5 C. The energy stored in the capacitor increases by ___

a) 250 J b) 2500 J

c) 2.5×10^{6} J d) 2.42×10^{-6} J

⇒ (b) $U = \frac{1}{2}\cdot\frac{Q^2}{C}$

$\therefore \Delta U = U_2 - U_1 = \frac{1}{2C}[Q_2^2 - Q_1^2]$

$= \frac{1}{2\times48\times10^{-6}}[(0.5)^2 - (0.1)^2]$

$= \frac{0.25-0.01}{96\times10^{-6}} = \frac{0.24}{96\times10^{-6}} = \frac{24\times10^{-2}}{96\times10^{-6}}$

$= 0.25\times10^{4} = 2500$ J

19) The earth has volume 'V' & surface area 'A'. What is the capacitance of the earth ?

a) $4\pi\epsilon_0\frac{A}{V}$ b) $4\pi\epsilon_0\frac{V}{A}$ c) $12\pi\epsilon_0\frac{V}{A}$ d) $12\pi\epsilon_0\frac{A}{V}$

⇒ (c) Capacitance of earth is

$C = \frac{Q}{V_1} = \frac{Q}{\frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{R}} = 4\pi\epsilon_0 R$ —(1)

In above expression, V is the potential of earth & not the volume.

Now, Volume of earth $(V) = \frac{4}{3}\pi R^3$

Surface area of earth $(A) = 4\pi R^2$

$$\therefore \frac{V}{A} = \frac{4\pi R^3}{3 \times 4\pi R^2} = \frac{R}{3}$$

$$\therefore R = \frac{3V}{A}$$

Put this in eqⁿ (1) ⇒

$$\therefore C = 4\pi\epsilon_0 \cdot \frac{3V}{A} = \frac{12\pi\epsilon_0 V}{A}$$

(22) What is the capacity of earth? (radius of earth = 6400 km, $4\pi\epsilon_0 = \frac{1}{9\times10^9}$)

a) 71.1 μF b) 711 μF c) 7 F d) 71 F

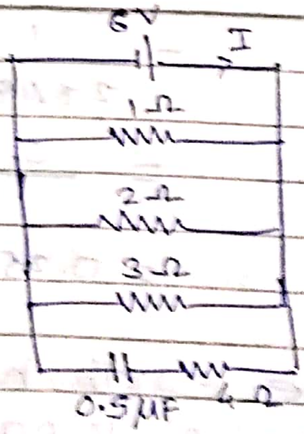
⇒ (b) Capacity of earth is

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{R}} = 4\pi\epsilon_0 R$$

$$= \frac{1}{9\times10^9} \times 6400\times10^3$$

$$= 711.11\times10^{-6}\,F = 711\,\mu F$$

(21) In given circuit, in the steady state the current through the battery & the charge on the capacitor resp. are



a) 2 A & 3 μC

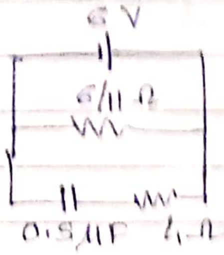
b) 11 A & 3 μC

c) $\frac{6}{11}$ A & $\frac{12}{7}$ μC d) zero ampere & 3 μF

⇒ (b) The resistances 1Ω, 2Ω & 3Ω are in parallel.

$\therefore \frac{1}{R} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6} = \frac{11}{6}$

$\therefore R = \frac{6}{11}\ \Omega$



In the steady state, the capacitor is fully charged & no current flows thro' it.
$\therefore$ No current flows thro' 4 Ω resistance.

$\therefore I = \frac{E}{R} = \frac{6}{6/11} = 11\ A$

& $Q = CV = 0.5 \times 10^{-6} \times 6$
$= 3 \times 10^{-6}\ C = 3\ \mu C$

(22) The capacity of a parallel plate condenser with dielectric constant 10 is 16×10^{-6} F. If the dielectric is removed, then the new capacity will be ____

a) 1.6×10^{-6} F b) 3.2×10^{-6} F
c) 0.8×10^{-6} F d) 2×10^{-6} F

$\Rightarrow$ (a) $K = \frac{C}{C_{air}}$

$\therefore C_{air} = \frac{C}{K} = \frac{16 \times 10^{-6}}{10} = 1.6 \times 10^{-6}$ F

(23) The capacity of a parallel plate condenser is 5 mF. When a glass plate is introduced between the plates of the condenser, its potential difference reduces to (1/8)th of the original value. The value of the dielectric constant of glass is ____

a) 6 b) 8 c) 4 d) 10

$\Rightarrow$ (b) $C_1 = \frac{Q}{V}$ & $C_2 = \frac{Q}{V/8} = \frac{8Q}{V}$ $\{\because$ P.D. reduces to $1/8^{th}$ V

$\therefore \frac{C_2}{C_1} = 2$ —— (1)

Also, $C_1 = \frac{A\epsilon_0}{d}$ & $C_2 = \frac{A\epsilon_0 K}{d}$

$\therefore \frac{C_2}{C_1} = K$ —— (2)

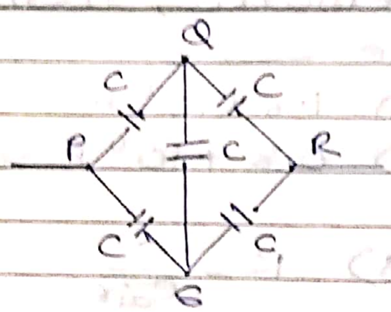
$\therefore$ From eq^n (1) & (2) $\Rightarrow$ $K = 2$

(24) The capacities of a parallel plate capacitor first with air & then on introducing oil between its plates are 50 μF & 120 μF resp. The dielectric constant of oil is —

a) 2 b) 4.2 c) 2.4 d) 0.48

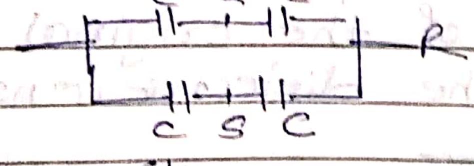
$\Rightarrow$ (c) $K = \frac{C}{C_{air}} = \frac{120}{50} = 2.4$

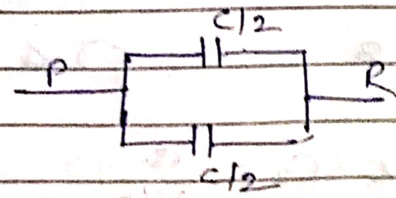
(25) Five identical condensers, each of capacity C are connected as shown in figure. The equivalent capacitance between P & R is —



a) C/2 b) C c) 2C d) 3C

$\Rightarrow$ (b) They form a Wheatstone bridge.
$\therefore$ Arm QS is ineffective.
$\therefore$ New ckt $\Rightarrow$





C & C in series

$\therefore \frac{1}{C_s} = \frac{1}{C} + \frac{1}{C}$

$= \frac{2}{C}$

$\therefore C_s = \frac{C}{2}$

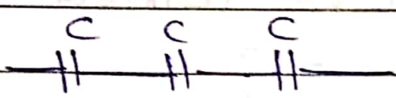
Now $\frac{C}{2}$ in parallel with $\frac{C}{2}$

$\therefore$ Resultant Capacity $= \frac{C}{2} + \frac{C}{2} = \frac{2C}{2} = C$

(26) You are given three condensers, each of capacitance 30 μF & a battery of emf 200 V. They can be joined in series or in parallel. Which arrangement of the condensers will give the minimum energy ?

a) All in series b) All in parallel
c) Two in series & third in parallel
d) Two in parallel & third in series.

$\Rightarrow$ (a) 1)

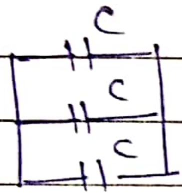


In series

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$\therefore C_s = \frac{C}{3} = \frac{30}{3} = 10\ \mu F$$

2)



In parallel

$$C_p = C + C + C = 3C$$
$$= 3 \times 30 = 90\ \mu F$$

Now, energy, $U = \frac{1}{2} C V^2$

$\because C_p > C_s$

$\therefore$ Series arrangement will give minimum energy.